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Predator Prey Models in Competitive Corporations

Rachel Von Arb
Olivet Nazarene University, rvonarb@olivet.edu

PREDATOR PREY MODELS IN COMPETITIVE CORPORATIONS

By

Rachel Von Arb

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Daniel L. Green	Daniel L. Green	3-13-13
Scholarship Project Advisor (printed)	Signature	Date
Charles W. Carrigan	Charles W. Carrigan	4/26/13
Honors Council Chair (printed)	Signature	Date
Janna R McLean	Janna R McLean	4/29/13
Honors Council Member (printed)	Signature	Date

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

ABSTRACT

Predator prey models have been used for years to model animal populations. In recent years they have begun to be applied to economic situations. However, the stock market has remained largely untouched. We examine whether the success of competitive corporations such as Target and Walmart, as measured by the indicators of price per share, market share, and volume, can be modeled by various predator prey models. We consider the basic Lotka-Volterra model and the two-predator, one-prey model, as well as a ratio-dependent model. We discuss the use of numerical techniques and regression analysis as tools to estimate model parameters. For Target and Walmart, the predator prey models mentioned above do not accurately fit the stock market data. In order to more fully explore the use of predator prey models in the stock market, we have examined several other competing companies using a simple Lotka-Volterra model, and found that critical model parameters were not statistically significant. While not statistically significant, these results help reinforce the unpredictability and complexity of markets and provide insight for future research.

Keywords: predator prey model, Lotka-Volterra, ratio-dependent, stage structured, time delayed, mathematical modeling, system of differential equations

INTRODUCTION

The purpose of this honors project is to determine whether the relationship between competitive corporations can be significantly modeled by a predator prey model. Predator prey models are mathematical models used by bio-mathematicians to describe relative population sizes of a predator and its prey over time. Previous studies have used predator prey models to analyze any number of economic situations, including but not limited to competition in the Korean stock market (Lee, Lee & Oh, 2005) and the competition between ballpoint and fountain pens (Modis, 2003). The simplest predator prey model used for this project is based on the Lotka-Volterra model, which is the most common of predator-prey models and relates one type of predator to one type of prey. The Lotka-Volterra model has since been expanded and modified in numerous ways to better model certain situations. In this paper we will also utilize a two-predator, one-prey model and a ratio-dependent model. Through this research we hope to provide insight into the competitive behavior of corporations as it relates to the competitive behavior of biological predators and prey.

SUMMARY OF PRELIMINARY RESEARCH

BASIC LOTKA-VOLTERRA PREDATOR PREY MODEL

The basic Lotka-Volterra model was proposed independently by the American mathematician Alfred Lotka and the Italian mathematician Vito Volterra in 1925 and 1926, respectively. The model includes a number of simplifying assumptions. First, the model assumes that the prey has an unlimited food supply. The second assumption is that the predator is the prey's only threat, and therefore any decrease in the prey population is related to predation. The next assumption is that the prey is the predator's only food supply, and that the predator's growth depends entirely on the amount of prey caught. Therefore, any increase in the predator population is related to predation. Additionally, we assume that the rate predators encounter prey is jointly proportional to the sizes of the two populations. This assumption of joint proportionality is represented by the terms pxy and dxy in the system of differential equations below. Finally, we assume that a constant proportion of encounters between predators and prey lead to prey death. With these simplifying assumptions, the Lotka-Volterra model can be constructed as a system of differential equations. Let us define the prey population at time t as $x(t)$, and the predator population at the same point in time as $y(t)$. Then $\frac{dx}{dt}$ represents the change in the prey population, x, as time t changes, and $\frac{dy}{dt}$ represents the change in the predator population, y, as time t changes. The basic Lotka-Volterra model is as follows:

$$\frac{dx}{dt} = bx - pxy$$

$$\frac{dy}{dt} = dxy - ry$$

In this system of differential equations, the parameter b represents the growth rate of the prey (species x) in the absence of interaction with the predator (species y). p and d are the parameters of the two interaction terms. p represents the effect of the predation of species y

on species x, while d is the growth rate of species y in perfect conditions: abundant prey and no negative environmental impact. Finally, r is the death rate of the species y from natural causes. Graphing this system of differential equations will yield a graph similar to that of Figure 1 below (taken from Beals, M., Gross, L., and Harrell, S., 1999):

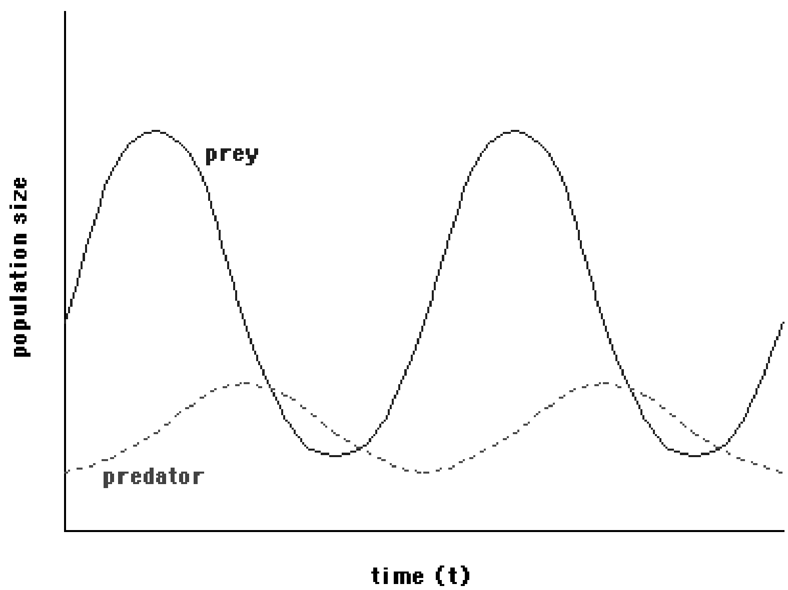

Figure 1: Simple Predator Prey Model

It is easy to see from this graph that a large enough increase in the number of predators leads to a decrease in the number of prey. This is logical from a biological standpoint, since a larger population of predators leads to increased interactions between predators and prey, and therefore increased prey death. This is also logical based on the Lotka-Volterra model. Looking again at the system of differential equations, we can see that an increase in the number of predators will lead to a decrease in $\frac{dx}{dt}$, the change in the prey population as time increases. As the prey population decreases, the predator population also begins to decrease, since the increased predator population can no longer be supported by the shrinking number of prey available. As the predator population decreases, the prey population begins to recover. Once the prey population has sufficiently recovered, the predator population once again increases. This brings us back to where we started: once again, an increase in the predator population leads to a decrease in the number of prey. This periodic pattern is common to all predator prey relationships.

The above graph is a time history, in which the sizes of the predator and prey populations are presented as functions of time. While a time history is fairly simple to understand, another important graph in predator prey modeling is the phase plane plot. The phase plane plot compares the population of predators to the population of prey, and is not dependent on time. Samples of phase plane plots created by MATLAB's ode23 and ode45 solvers are depicted below in Figure 2 (taken from *Numerical Integration of Differential Equations*):

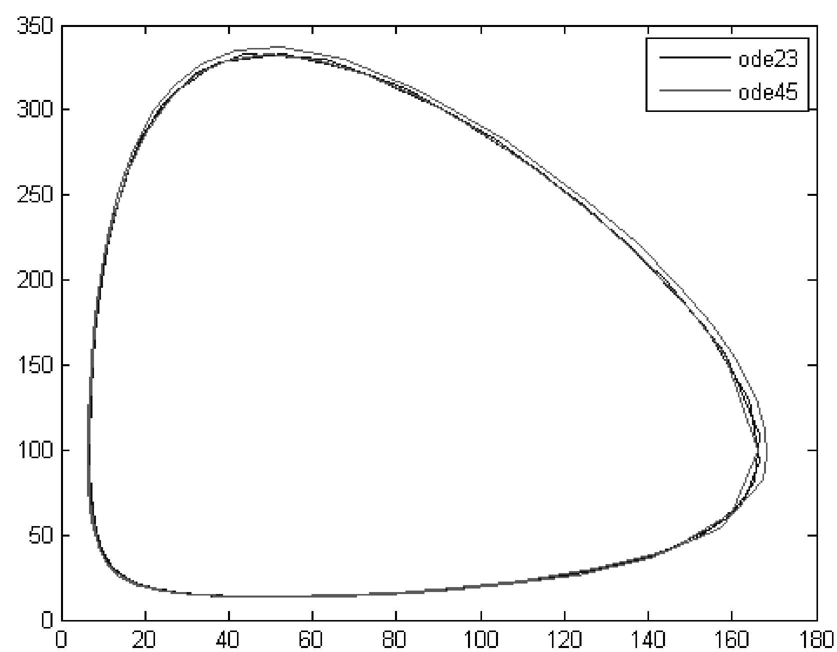

Figure 2: Predator Prey Phase Plots

As can be seen above, the ode23 and ode45 solvers have slightly different plots. The ode45 solver's plot is slightly smoother than that of the ode23 solver. This difference lies not in the data, but in the programming of the two solvers. As can also be seen, phase plane plots relate the size of the predator population to the size of the prey population, which in a predator prey relationship creates a rounded plot. This is because the predator population increases shortly after the prey population increases, causing the prey population to decrease, and quickly causing the predator population to decrease as well. At this point, the cycle has returned to where it started and begins again. Thus, the periodic pattern evident in the graph in Figure 1 above is also evident in Figure 2 as the rounded plot seen above.

The classic example of a Lotka-Volterra type predator prey model is the relationship in population sizes of the Canadian lynx and snowshoe hare over 200 years ago. Canadian lynx have natural prey besides the snowshoe hare, but rely on the snowshoe hare as their primary prey. The data for this example come from a century of pelt trading records collected by the Hudson's Bay Company, which was heavily involved in the pelt trading business. The data reveal that the relationship between the populations of these two species over time is well modeled by a predator prey model, as seen in Figure 3 below (taken from *Predator Prey Models,* 2000):

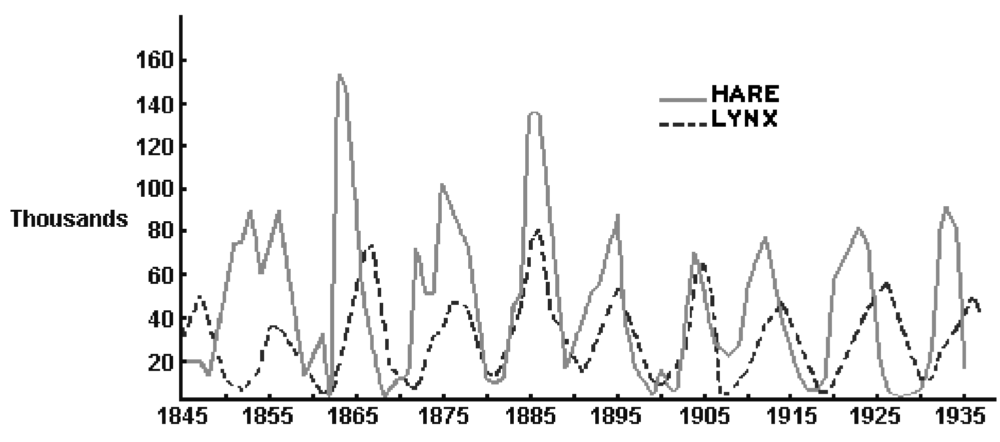

Figure 3: Hare and Lynx Predator Prey Model

As expected, as the lynx population increases the hare population begins to decrease, which soon leads to a decrease in the lynx population as well. This decrease in lynx population size, as predicted, leads to an increase in the size of the hare population, which quickly leads to an increase in the size of the lynx population. It is important to note that the hare-lynx graph is less smooth than the generic predator prey graph; this is because the hare-lynx graph only contains data from certain points in time and consists of real data over time, while the generic graph above was continuous and consisted of ideal data.

TWO-PREDATOR, ONE-PREY MODEL

The two-predator, one-prey model is a variation on the basic Lotka-Volterra predator prey model that accounts for a situation in which two predator populations are present and both predate on a single prey species as their primary food source. A two-predator, one-prey system is composed of three differential equations. One differential equation represents the change in population size over time for each of the populations. In this case, let us define the prey population at time t as $x(t)$, the first predator's population at the same time as $y(t)$, and the second predator's population at time t as $z(t)$. The simplest system of equations modeling this type of behavior is as follows:

$$\frac{dx}{dt} = ax - bxy - cxz$$

$$\frac{dy}{dt} = dxy - ey$$

$$\frac{dz}{dt} = fxz - gz$$

In this case, the population of the prey, species x, increases in perfect conditions at a rate of a, and decreases in response to predation from both species y and species z (this is where the – bxy and – cxy interaction terms come from in the first equation in the system). The populations of the two predator populations, species y and z, have death rates of e and g, respectively, and grow in response to predation on species x at rates of d and f, respectively. Note that e and g are not necessarily the same: the predators do not necessarily have the same death rate. Additionally, d and f are not necessarily the same: the effect of predation on both of the predator populations may be different.

A system with two predators and one prey population can have many different end results, or equilibria. In one case, the first predator is much more effective than the other predator at catching prey, causing the second predator to eventually become extinct. In another case, both predators are equally skilled at catching prey, and at the system's equilibrium both predator populations are still present. In a third case, both predators may become extinct, causing the prey population to grow freely. The state of equilibrium of a system depends on the parameter values (a, b, c, d, e, f, and g) and the initial conditions (that is, $x(0)$, $y(0)$, and $z(0)$, the initial populations of the prey and both predators).

RATIO-DEPENDENCE, STAGE-STRUCTURING, AND TIME DELAYS IN PREDATOR PREY MODELS

Ratio-dependence, stage-structuring, and time delays are three common ways of making a predator prey model more realistic. These methods each eliminate one of the simplifying assumptions made by the basic Lotka-Volterra model. However, this comes at a cost. Because these methods eliminate simplifying assumptions, they also greatly increase the complexity of the model.

Ratio-Dependent Models

As can be seen, the basic Lotka-Volterra model assumes that the rate of predation depends entirely on the prey population at a given time. The ratio-dependent model adapts this by basing the rate of predation on both prey and predator population densities. According to many recent biologists, the use of ratio-dependence makes the basic model more realistic. The system of equations for a ratio-dependent model is as follows:

$$\frac{dx}{dt} = x(a - bx) - \frac{cxy}{my + x}$$

$$\frac{dy}{dt} = \frac{fx}{my + x} - ry$$

This may appear to come out of thin air, but can in fact be easily explained. There are three types of Holling functional responses, all of which relate the rate of food intake by a predator to the population of the prey. A linear, or Holling type I functional response, is used in the Lotka-Volterra predator-prey model. The terms in a ratio-dependent model are based on a Holling type II functional response:

$$f(R) = \frac{cR}{m + R}$$

in which R is replaced by $\frac{x}{y}$ in order to account for the ratio of predator to prey. The Holling type II ratio-dependent functional response thus becomes

$$f\left(\frac{x}{y}\right) = \frac{c\left(\frac{x}{y}\right)}{m + \left(\frac{x}{y}\right)} = \frac{cy}{my + x}$$

which, when inserted into a basic predator-prey model, gives the system of equations above.

Stage Structured Models

Stage structure makes a model more realistic by assuming different vital rates (survival rates and birth rates) based on age. For instance, in most populations the most susceptible members of a population are the very young and the very old. Additionally, both very young and very old members of a population have low reproductive contributions. While this model is useful for many biological models, we will not be considering it in our market analysis.

Time Delayed Models

Finally, time delayed predator prey models relate current rates of growth or decay to previous population sizes. This is a concept best explained by example. In time delayed predator prey models, the effect of predation on species x can relate to the size of population y at a previous point in time in order to account for the fact that only mature predators hunt. Another case of delay is delaying the growth rate of species y in perfect conditions to account for gestation and maturation. Again, these adjustments to the model eliminate some simplifying assumptions, making the model more realistic, but also more complicated. We have not adjusted our model for time delay.

PREVIOUS PREDATOR PREY RESEARCH IN ECONOMICS

Although there has been little research into the role of using predator prey models to model relations between specific companies, there has been much research into the role of predator prey models in the field of economics. A fundamental model in economics is the Goodwin model, which attempts to model economic fluctuations in general by relating real wages and real employment. The Goodwin model can easily be related to the Lotka-Volterra model, which Vadasz (2007) does in his paper. A more concrete example of predator prey models in the economic field, and one that is especially interesting and relevant, is found in the research of Seong-Joon Lee, Deok-Joo Lee, and Hyung-Sik Oh (2005) into the dynamics of the Korean Stock Exchange (KSE) and the Korean Securities Dealers Automated Quotation (KSDAQ), two competing Korean stock markets. According to research, the KSE played the role of prey to the KSDAQ, until eventually the two markets stabilized into a pure competition relationship. In his paper, Theodore Modis (2003) discusses the relative success of fountain pens compared to ballpoint pens from 1929 to 2000. In this research, the two types of pens initially followed a predator-prey model, but no longer interact today. Research by Edward Gracia (2004) fits the business cycle to the Lotka-Volterra predator prey model. Interestingly, his results were compatible with the efficient markets hypothesis. As can be easily seen, there are any number of important applications of predator prey models in the economic spectrum. However, predator prey models have rarely been used to estimate the behavior of individual companies.

METHODS

All data sets used are publicly accessible. Unless otherwise stated, all data are taken from Yahoo! Finance (www.finance.yahoo.com). Curve-fitting this data to a predator-prey type model requires the use of the regression techniques explained below in order to estimate parameters. For linear regressions, we utilized Excel to initially estimate parameters, but statistical packages such as SPSS can be used as well. Numerical computation software such as MATLAB can be used to estimate parameters of differential equations directly. Once the data were fitted, statistical analyses were performed to determine whether the results were significant. For models that are significant, it is possible to perform equilibrium analysis.

PARAMETER ESTIMATION TECHNIQUES

The least squares method is typically used to fit data to a polynomial function. This method works by assuming that the best fit curve minimizes the sum of the squares of the differences between the fitted curve and the data points. This gives a much larger penalty for larger differences between the fitting function and the data: for example, a difference of 1 adds 1 to the sum of squares, while a difference of 2 adds 4 to the sum. This is the most common form of curve fitting, and can be used for higher-order polynomial functions. We use Excel regressions, which utilize the least squares method, to initially approximate our parameters.

MATLAB

It is possible to find more precise numerical solutions to a given set of differential equations using various methods in MATLAB. The Runga-Kutta methods, used by MATLAB's ode23 and ode45 solvers, are based on the Taylor series methods, and are frequently used in systems of ordinary differential equations to estimate the values of an equation at a particular point. The Taylor series methods themselves are based on the Taylor series representation of equations. The Taylor series for a continuous function $x(t)$ with infinitely many continuous derivatives is a series of the form

$$x(t+h) = x(t) + hx'(t) + \frac{1}{2!}h^2x''(t) + \frac{1}{3!}h^3x'''(t) + \ \dots + \frac{1}{m!}h^m x^{(m)}(t) + \ \dots$$

As can be seen by the formula, the Taylor series is an infinite series. The Taylor series methods approximate $x(a+h)$ by using a truncated version of the Taylor series listed above. The Runga-Kutta methods are similar to the Taylor series methods, but require none of the differentiation required by Taylor series methods to find an approximation at a particular value. MATLAB's ode23 and ode45 solvers use these methods to find the numerical solution to a given system of differential equations at requested points, which can be used to plot a graph of the numerical solution to the system. Using this information, basic statistical analysis can be performed on the system of differential equations fitted to the data to determine how well the curve fits as compared to other models.

Statistical Software

For the basic Lotka-Volterra model and the two-predator, one-prey model, it is possible to attain a rough approximation of the parameters through Excel or SPSS before running the data through an ordinary differential equation solver in MATLAB. We discuss this process in-depth for individual examples in the sections entitled Target and Walmart: Basic Lotka-Volterra Model and Target and Walmart: Two-Predator One-Prey Model. Essentially, we simplify the equations to linear models with one dependent variable and either one or two dependent variables. We then use Excel regressions to approximate parameters and determine whether a model is promising enough to use MATLAB to further estimate parameters for the data. In order for Excel results to be considered significant enough, we impose two conditions: the Excel parameter approximation should have $p < .05$, and the interaction term must be non-zero. If $p < .05$, the model is statistically significant. If the interaction term is zero, this implies no interaction between the two populations, and thus the model is not truly an interactive predator prey model. If either of these conditions are not met, we examine a different date range until we find a significant result. Let us examine a sample Excel output:

SUMMARY OUTPUT	Target (Prey)							
Regression Statistics			b =	0.03006				
Multiple R	0.0292		p =	0.00000				
R Square	0.0009							
Adjusted R Square	-0.0040							
Standard Error	0.2444							
Observations	206							
ANOVA								
	df	*SS*	*MS*	*F*	*Significance F*			
Regression	1	0.0104	0.0104	0.1740	0.6770			
Residual	204	12.1823	0.0597					
Total	205	12.1926						
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.0301	0.0633	0.4749	0.6354	-0.0948	0.1549	-0.0948	0.1549
X Variable 1	0.0000	0.0000	-0.4171	0.6770	0.0000	0.0000	0.0000	0.0000

Table 1: Sample Target Regression

SUMMARY OUTPUT	Walmart (Predator)							
Regression Statistics			d = 0.00000					
Multiple R	0.0429		r = 0.01255					
R Square	0.0018							
Adjusted R Square	-0.0031							
Standard Error	0.1618							
Observations	206							
ANOVA								
	df	*SS*	*MS*	*F*	*Significance F*			
Regression	1	0.0098	0.0098	0.3757	0.5406			
Residual	204	5.3375	0.0262					
Total	205	5.3473						
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-0.0125	0.0214	-0.5859	0.5586	-0.0548	0.0297	-0.0548	0.0297
X Variable 1	0.0000	0.0000	0.6130	0.5406	0.0000	0.0000	0.0000	0.0000

Table 2: Sample Walmart Regression

The standard Excel output consists of everything seen above except the yellow and blue highlighted portions, which have been added to each regression in this paper for clarity. The yellow cell lists the corporation and the role it plays in the predator-prey model. The blue cells give the parameter estimations, which are based on the green cells. Recall that we require the interaction term coefficients, p and d, to be non-zero in order to refine our estimates using MATLAB. Finally, the purple cells give the p-value of the model. Recall that we require $p < .05$, at which point the model is statistically significant, in order to use MATLAB to refine our estimates.

ANALYSIS OF SIGNIFICANCE OF RESULTS

As stated before, Excel results will be considered significant if they meet the following two conditions: $p < .05$ and the interaction term coefficients are non-zero. If Excel results are significant, we will run the data for the same dates through MATLAB to determine more precisely estimated parameters. These MATLAB results will be considered against a linear fit and a quadratic fit of the same data to determine whether a predator-prey model is a better fit than either of these methods. This will be determined by the F-statistic of the model. Even if a predator-prey model is the best fitting model of the three, results will only be considered significant if $p < .05$ and the interaction term coefficients are non-zero.

RESULTS

TARGET AND WALMART

Graphical Analysis

Before utilizing MATLAB or any other processing software to estimate parameters, it is important to analyze the data graphically. During the examination, we checked to see if the data resembled a predator-prey model such as those discussed previously. For a simple one-predator one-prey model, the graphs of two related companies over time should appear periodic and the graph of the "predator" corporation should lag behind that of the "prey" corporation, as seen previously in Figure 1. In the Target and Walmart data, we first examined the unadjusted volume data. In this data set, Target appeared to lag Walmart slightly. However, the unadjusted data included large rises and falls due to outliers. In order to remove these extreme points, we used 3 day, 7 day, 50 day, and 200 day moving averages. Moving averages assign the average value over a series of days to one particular day. This smooths the data, removing some of the large variability. Graphs of the 7 day, 50 day, and 200 day moving averages can be seen in Appendix A: Graphs. Unfortunately, the moving averages appear to eliminate the lag between the Target and Walmart data. As can be seen, the 200 day moving average eliminated a large amount of variability and lag. For the sake of completeness, a graph of the monthly volumes for both Target and Walmart has also been included in Appendix A: Graphs. The 7 day moving average appeared to best represent the periodic qualities of the graph, while removing large outliers. Additionally, we chose to work with the data between late 1998 and late 2004, as the data in this time range appear to have a minimal trend line, if any. This is important because the predator-prey models we are examining do not account for a trend, which is a general increase or decrease in the dependent variable over time.

Basic Lotka-Volterra Model

After estimating the parameters on this set of data, using Target as the prey and Walmart as the predator, we found that the interaction terms for the one-predator, one-prey model are calculated as zero, and, additionally, the model is not significant.

We used a basic estimation of the parameters to determine whether or not the data were significant. Let us discuss how the basic estimates of the parameters of the one-predator, one-prey model were found. As seen previously, the one-predator, one-prey model has the form

$$\frac{dx}{dt} = bx - pxy$$

$$\frac{dy}{dt} = dxy - ry$$

Looking at the equation for dx/dt, by dividing both sides by x, we attain a linear equation of one variable:

$$\frac{1}{x}\frac{dx}{dt} = b - py$$

To determine dx/dt, we could use the approximation

$$\frac{dx}{dt} \approx \frac{f(x+h) - f(x)}{h}$$

However, a much better approximation can be found by using the approximation

$$\frac{dx}{dt} \approx \frac{f(x+h) - f(x-h)}{2h}$$

This approximation is much better than the previous approximation since it is $O(h^2)$, rather than $O(h)$. There are also higher order methods, but we did not use these for the initial approximation since such precision is not necessary for an initial estimation. This gave a simple linear model, with independent variable y and dependent variable $\frac{1}{x}\frac{dx}{dt}$. The parameters b and p were easily estimated by a simple linear regression using Excel. Similarly, we estimated the parameters r and d using a modified version of the second equation, which related $\frac{1}{y}\frac{dy}{dt}$ to x.

We found that for the current data, the interaction terms for the one-predator, one-prey model are zero. This implies that Walmart's success does not impact Target's success, and vice-versa. This can be seen graphically in the charts in Appendix A: Graphs. This can also be seen statistically in the insignificant p-values of .6770 and .5406 found in the regressions based on the Target and Walmart monthly stock volume data from May 2, 1983 to April 2, 2001, which are found in in Appendix B: Regressions.

Two-Predator One-Prey Model

We next considered a two-predator, one-prey model. For this model, we used Target and Walmart's monthly stock volume as the two predator populations, and used the S&P 500's monthly stock volume as the prey population. This is reasonable, since Target and Walmart are both competing for consumers, while the S&P 500 is a readily accessible indicator of consumer spending. We performed a graphical analysis on these data similar to the one performed for the one-predator, one-prey model. In this case, it appeared that data between January 1988 and December 2000 was the most promising for being modeled by a two-predator, one-prey model. However, for the sake of mathematical completeness, we tested data from January 2007 to the present, from April 1983 to the present, and from May 1983 to April 2001 as well. We next estimated parameters.

Again, we used a simple estimate of the parameters. Recall that the equations for a two-predator, one-prey model are

$$\frac{dx}{dt} = ax - bxy - cxz$$

$$\frac{dy}{dt} = dxy - ey$$

$$\frac{dz}{dt} = fxz - gz$$

The parameters d, e, f, and g could be estimated as before. However, the parameters a, b, and c were modeled using multiple regression. This was again performed in Excel. The regression for all data from May 2, 1983 to April 2, 2001 is shown in the Excel outputs in Appendix B: Regressions. As can be seen there, this model is not significant for this time range; we did not find significant results in any of the time ranges we tested. Additionally, we found that the interaction parameters for this model (b, c, d, and f) were zero.

Ratio-Dependent Model

Recall that a ratio-dependent model has the form

$$\frac{dx}{dt} = x(a - bx) - \frac{cxy}{my + x}$$

$$\frac{dy}{dt} = \frac{fx}{my + x} - ry$$

Unlike the Lotka-Volterra model and the one-predator, one-prey model, there is no trick to approximate the parameters of this model before using MATLAB to refine the parameter estimations. Therefore, we used MATLAB directly from the data in order to estimate parameters. The MATLAB program to do so is similar to those used for the previous models, except in this case the program used followed the system of equations used for a ratio-dependent model. Using the Target and Walmart data based on graphical analysis, as before, we did not find a ratio-dependent model that fit well.

Detrended Data

As mentioned before, our models did not account for any increases or decreases in the data over time. In order to remedy this and attempt to model Target and Walmart's successes within the 2003 to 2012 range, we first detrended the data. We fitted a simple linear model of the form $y = mx + b$ directly to the data and determined the estimated value, $\hat{y}(t)$, at each time. Our new points were the original value minus the estimate of the value found using the linear model:

$$y'(t) = y(t) - \hat{y}(t)$$

where $y'(t)$ is our detrended data point, $y(t)$ is our original data point, and $y(t)$ is our estimate using a linear model. This accomplished two things: it centered the data about zero, and it eliminated any trend. For example, using the data from 4/1/2009 to 5/1/2012, the detrended data as compared to the original data is as follows:

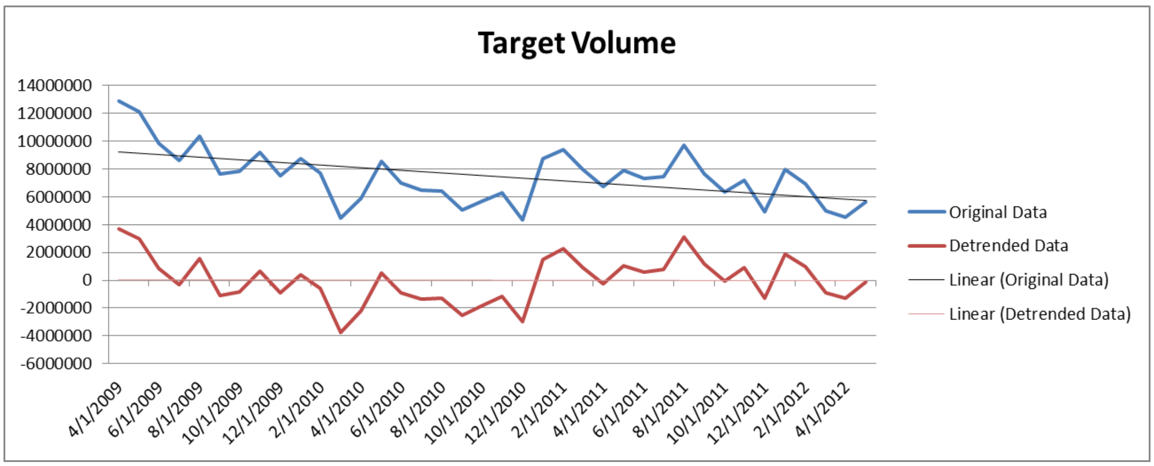

Figure 4: Detrended Target Data, 2009-2012

As can be seen, the trend line of the detrended data became the line $y = 0$, which eliminated the downward trend in the original data.

Using detrended data, we estimated our predator-prey model parameters as before. Once again, we found the interaction term coefficients were zero, and the models were not significant. The regressions for the detrended data are found in Appendix B: Regressions.

OTHER CORPORATIONS

We also considered using different data with a one-predator, one-prey model. We examined Apple (APPL) versus Dell (DELL) and Apple versus Microsoft (MSFT), as well as Dell versus Microsoft. Additionally, we tested Caterpillar (CAT) versus the S&P 500 (GSPC) and versus the NASDAQ Composite (IXIC). We also considered the Dow Jones U.S. Oil & Gas Index (DJUSEN) versus the Dow Jones U.S. Coal Index (DJUSCL). Finally, we considered alternative fuel prices, as represented by solar thermal prices (data from www.eia.gov) versus natural gas prices (data from www.eia.gov). For each comparison, various time periods were tested based on graphical analysis of each separate comparison, as in the Target and Walmart data we first considered. However, no examined comparison produced a significant result. The data from each comparison are included in graphical form in Appendix A: Graphs, and one regression from each comparison is included in Appendix B: Regressions.

DISCUSSION

DISCUSSION OF RESULTS

Recall from before that in order for us to consider Excel results to be significant, we imposed two conditions: $p < .05$, and a non-zero interaction term. If $p < .05$, the model is statistically significant. However, even if the model is statistically significant, if the interaction term is zero there is no interaction between the two populations, and thus the model is not an interactive predator prey model. We do not meet both of these conditions for any of the data examined, and therefore we consider all attempts at modeling so far unsuccessful. For the date ranges tested, this implies that Walmart's success does not impact Target's success and vice-versa. Similarly, for the other companies compared, we have not found a significant impact from the success of one company on the success of another company. However, we cannot conclusively say that there is no market situation for which a predator-prey model will fit the data; in fact, this would be false, as seen in the research of Seong-Joon Lee, Deok-Joo Lee, and Hyung-Sik Oh on the dynamics of the Korean Stock Exchange (KSE) and the Korean Securities Dealers Automated Quotation (KSDAQ), which we examined earlier in this paper. We also cannot say that there is conclusively no predator prey type model which significantly represents the success of Target and Walmart, or any of the other corporations tested. We can only say that we have not yet found a predator-prey model to significantly model individual companies or industries within the market.

Previous research into using predator-prey models in the stock market has been successful, likely because the data being modeled were simpler in nature. In the research by Lee, Lee, and Oh on the Korean Stock Exchange and the Korean Securities Dealers Automated Quotation, the markets were much smaller and likely more isolated than the American market. The KSE and the KSDAQ are two major exchanges with little other competition, whereas the corporations we have looked at are not the sole competitors in their given market sector. Therefore, the KSE and the KSDAQ follow more of the simplifying assumptions of a basic predator-prey model and have less confounding variables to complicate the data. In the research presented by Theodore Modis (2003) on the competition between fountain pens and ballpoint pens, there were again only two competitors involved. This competition also occurred in a very specific portion of the market, before computerized trading and the success of the internet, which limited the impact of outside factors on the data.

If Target and Walmart's success can be modeled by a predator prey model, there are many possible contributing factors to our inability to find such a model. Basic predator prey models such as the Lotka-Volterra model and the simple two-predator, one-prey model contain many simplifying assumptions, making them simultaneously easier to work with and less realistic. We eliminated one of these assumptions by considering a ratio-dependent model, but there are many other simplifying assumptions that we simply did not have the time to consider. It is likely that one or more of the model's simplifying assumptions is violated by the stock market data we have been working with. For this reason, more complex models such as stage structured models and time delayed models may better fit the data.

Additionally, when dealing with real data, it is always important to consider outside confounding factors. In this case, the stock market is extremely sensitive to small changes that are not accounted for in the model. For example, the housing crisis greatly increased stock volume for both Target and Walmart in a way that cannot be accurately modeled by a predator prey model. We adjusted for the housing crisis by excluding this time frame while estimating parameters and by examining detrended data. Though we considered large outside factors, there are many additional factors that can affect stock volume, such as smaller economic fluctuations, additional market competitors, and even day of the week. Additionally, because we are working with market data, even if the data are able to be modeled by a predator prey model, we will have random errors, and large outliers may skew parameter estimations.

Because of a combination of the factors discussed, it is possible that stock market data cannot be readily modeled by predator prey models, except in specific circumstances. Based on previous results, these circumstances may include the following: the modeled corporations being the sole competitors in a given market sector, and isolation of the system from economic confounding variables. These circumstances address the assumptions in the basic Lotka-Volterra model that the predator is the prey's only threat, that the prey is the predator's only food supply, and that there is no negative environmental impact on the predator. We recommend that others considering predator-prey models in the stock market take these factors into consideration when selecting corporations.

DIRECTION OF FURTHER RESEARCH

Despite the fact that the data we have examined do not easily follow a predator-prey model, there remain many possibilities that have not been considered. For instance, in the two-predator one-prey model it is possible to use NASDAQ stock volumes instead of the S&P 500's as a possible prey indicator, or to use data besides stock volume as indicators of success. There are also many corporations that have not been tested. For example, it would be very interesting to test Walmart's sales in a small town versus the sales of a small family owned business in the same town. We recommend choosing corporations that are the sole competitors in a given market area, and that have some isolation from the general economy in order to follow the assumptions of the model. Finally, we have not considered a stage structured or a time delayed model for the data we did examine, which would eliminate some simplifying assumptions. In these examples, it is still possible to estimate parameters using Excel or SPSS for the basic Lotka-Volterra and for the two-predator one-prey models. After these parameters have been estimated, the approximations of the predator and prey populations at a given time as $\hat{x}$ and $\hat{y}$ should be used to perform a statistical analysis of the significance of the model. Models should be considered to be significant at probability $p < .05$ if the interaction terms are non-zero. If any competitive corporations examined can be modeled by a predator-prey model, the long-term equilibrium behavior of the system should be analyzed.

CONCLUSION

Predator-prey models are extremely interesting and versatile, and have been used in the past to model diverse situations. Many of these situations involve neither predator nor prey. While predator-prey models have been used to model various economic situations, their application to the stock market has been scarce. In this paper we examined various stock market data by using a basic Lotka-Volterra model, a two-predator, one-prey model, a ratio-dependent model, and by using detrended data. We hope with this paper to encourage continued research into the area of modeling specific companies over time. While the current results are not statistically significant, they reinforce the unpredictability and complexity of markets and provide insight for future research.

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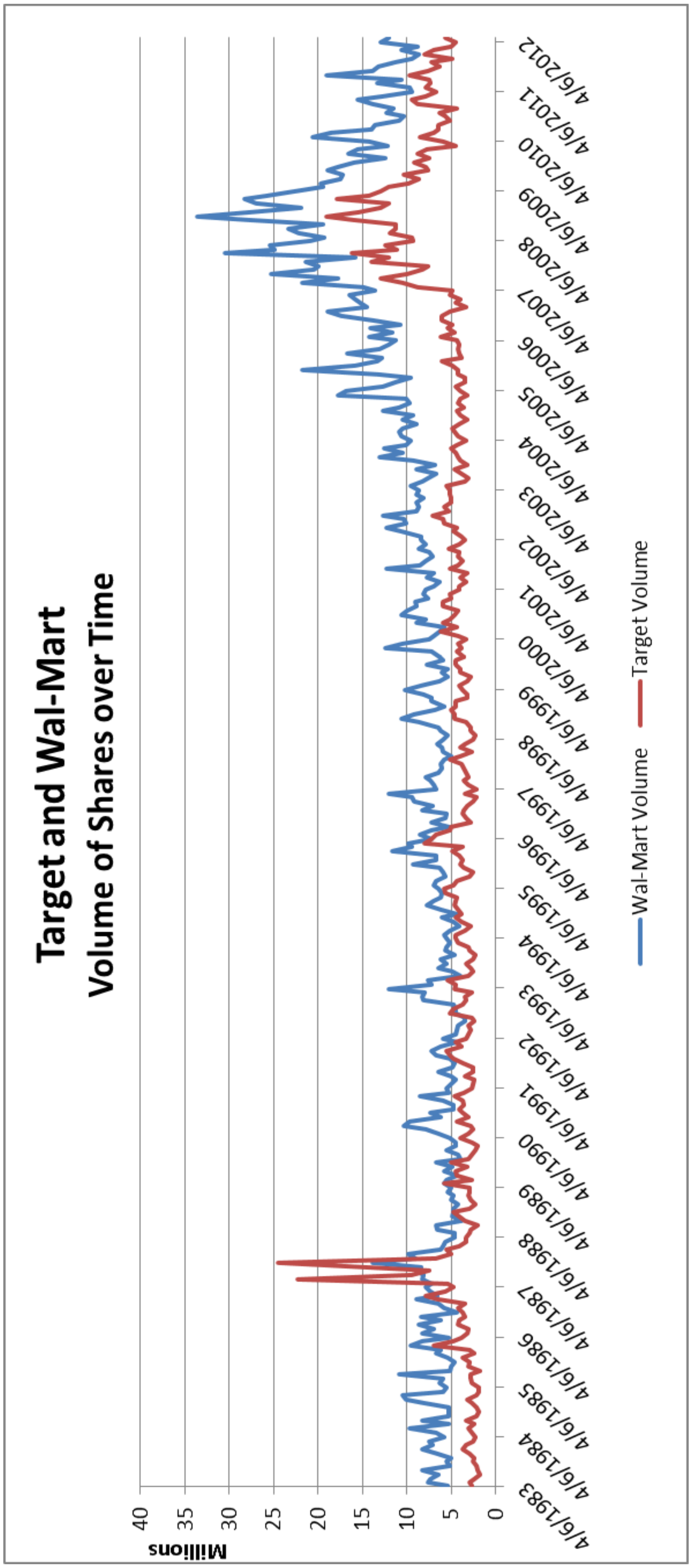
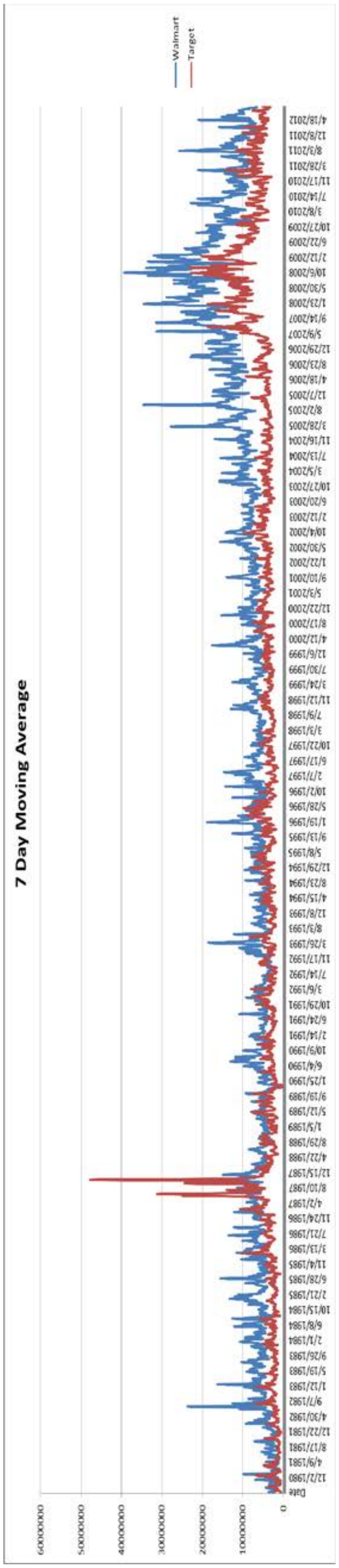
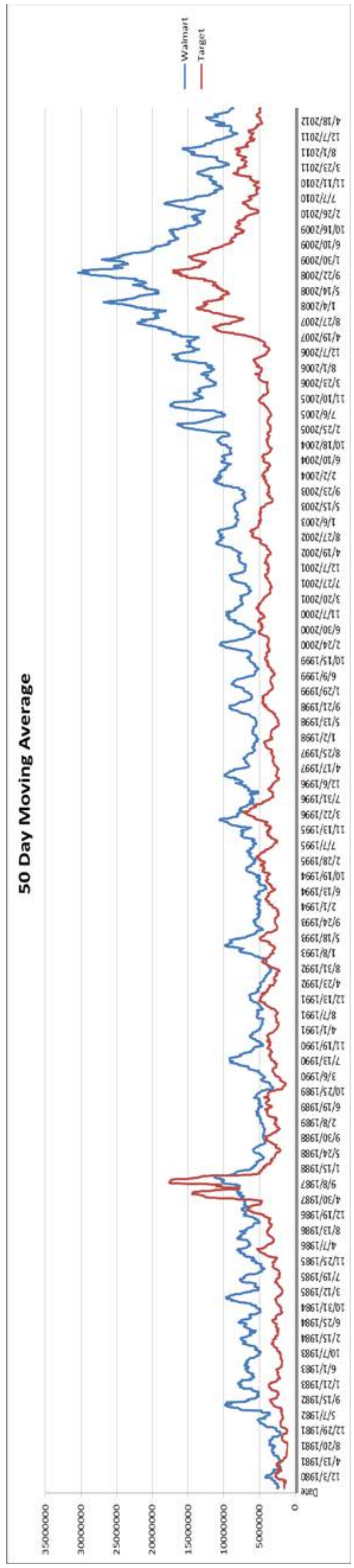
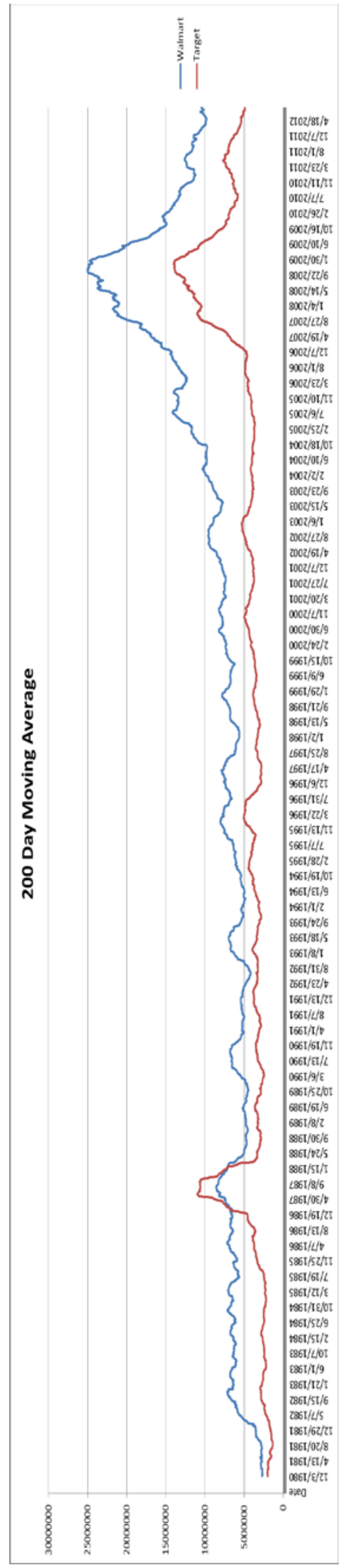
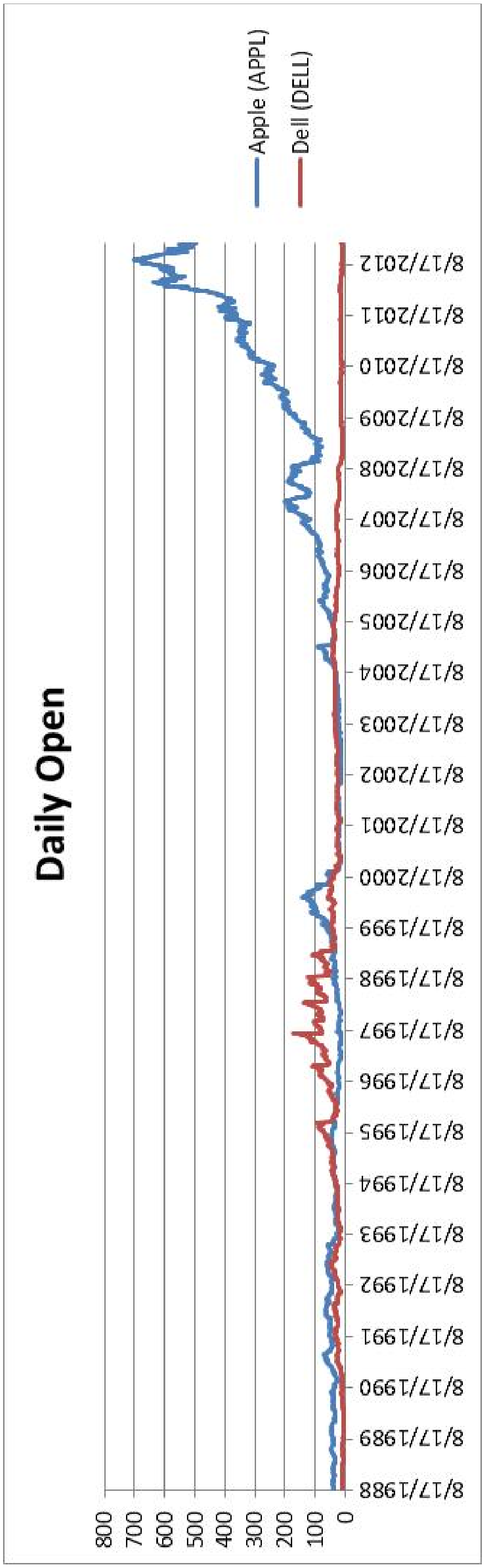
APPENDIX A: GRAPHS

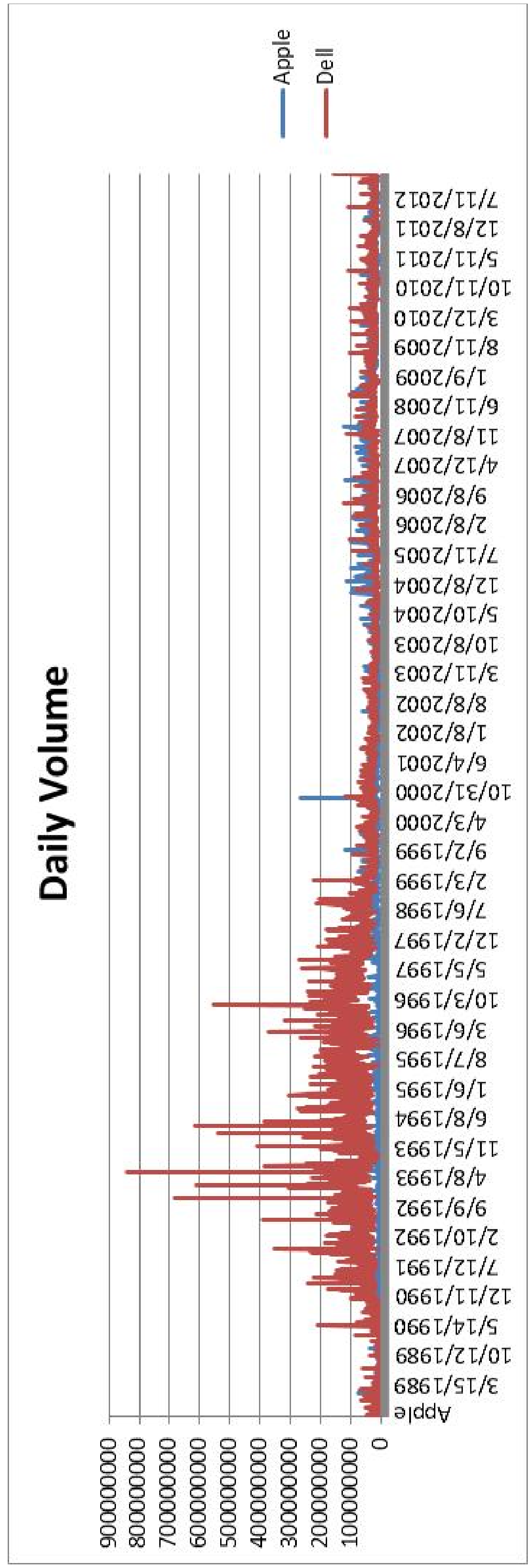


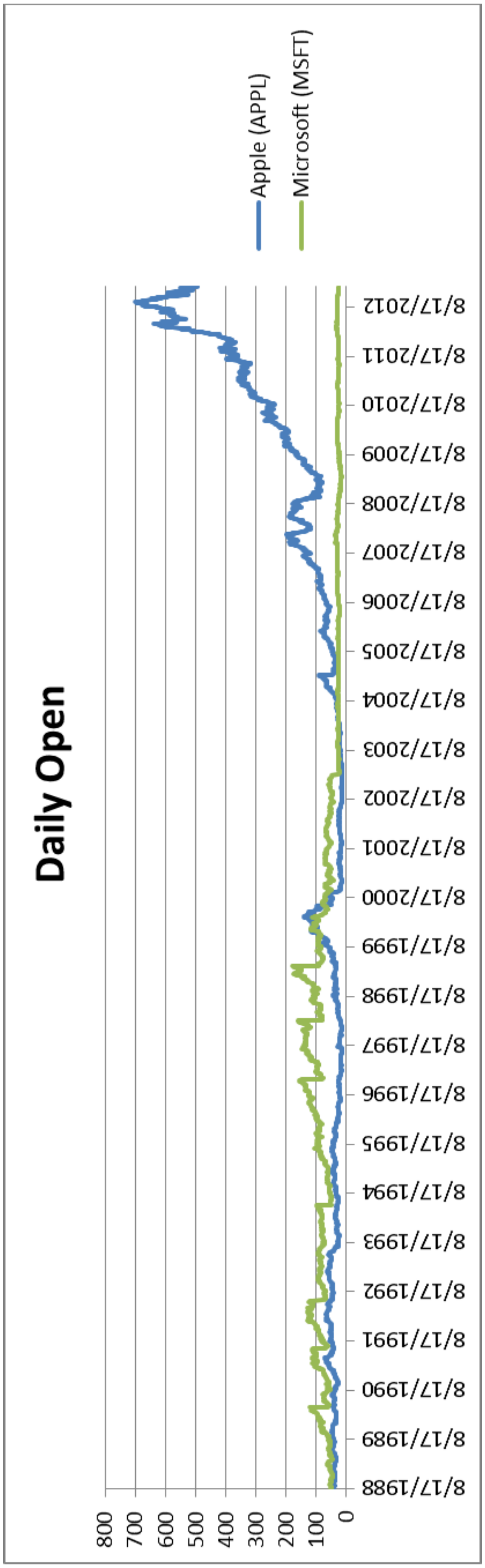

7 Day Moving Average
Walmart
Target
Date


50 Day Moving Average
Walmart
Target
Date


200 Day Moving Average
Walmart
Target
Date


Daily Open
Apple (APPL)
Dell (DELL)


Daily Volume
Apple
Dell


Daily Open
Apple (APPL)
Microsoft (MSFT)
800
700
600
500
400
300
200
100
0
8/17/1988
8/17/1989
8/17/1990
8/17/1991
8/17/1992
8/17/1993
8/17/1994
8/17/1995
8/17/1996
8/17/1997
8/17/1998
8/17/1999
8/17/2000
8/17/2001
8/17/2002
8/17/2003
8/17/2004
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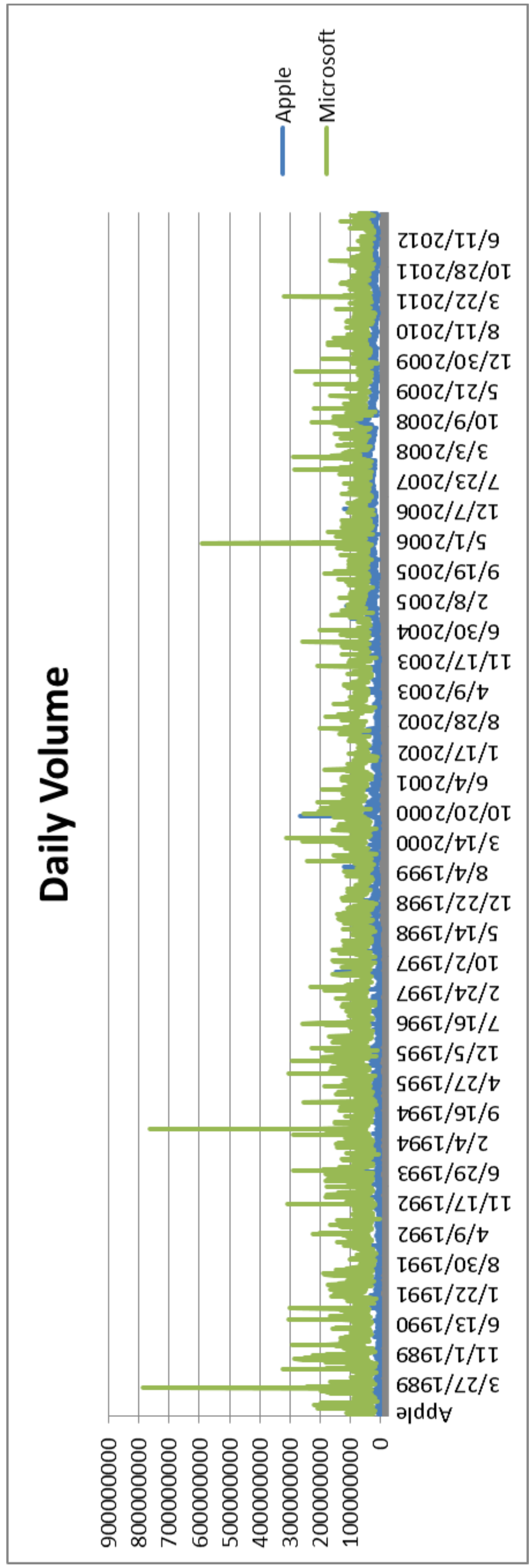
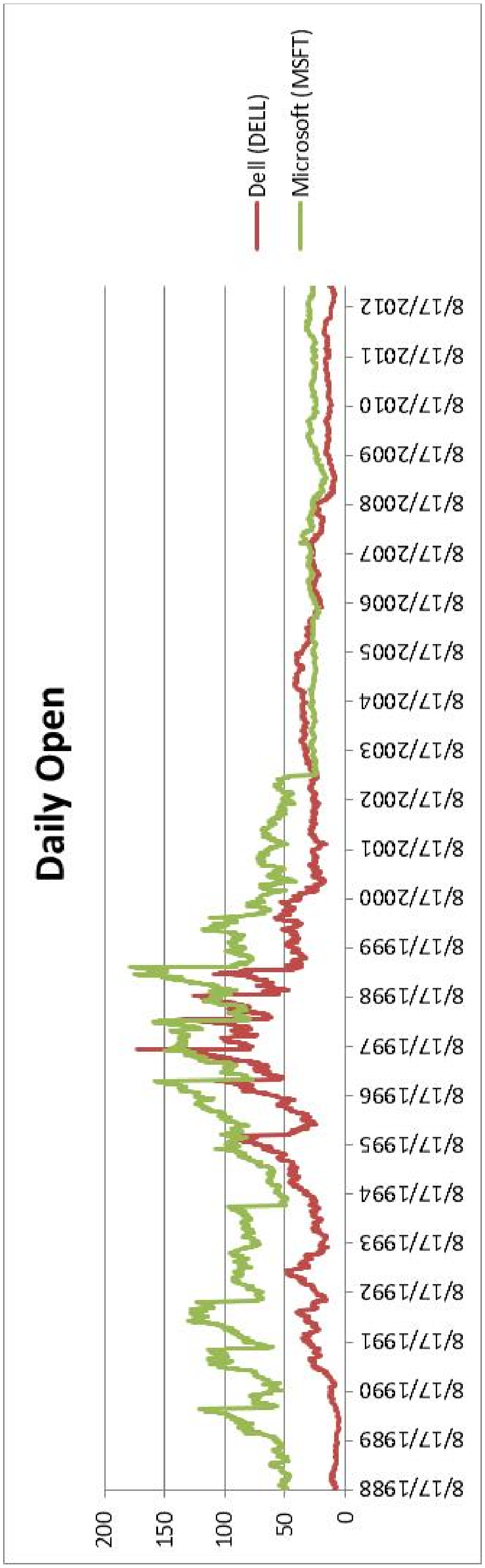
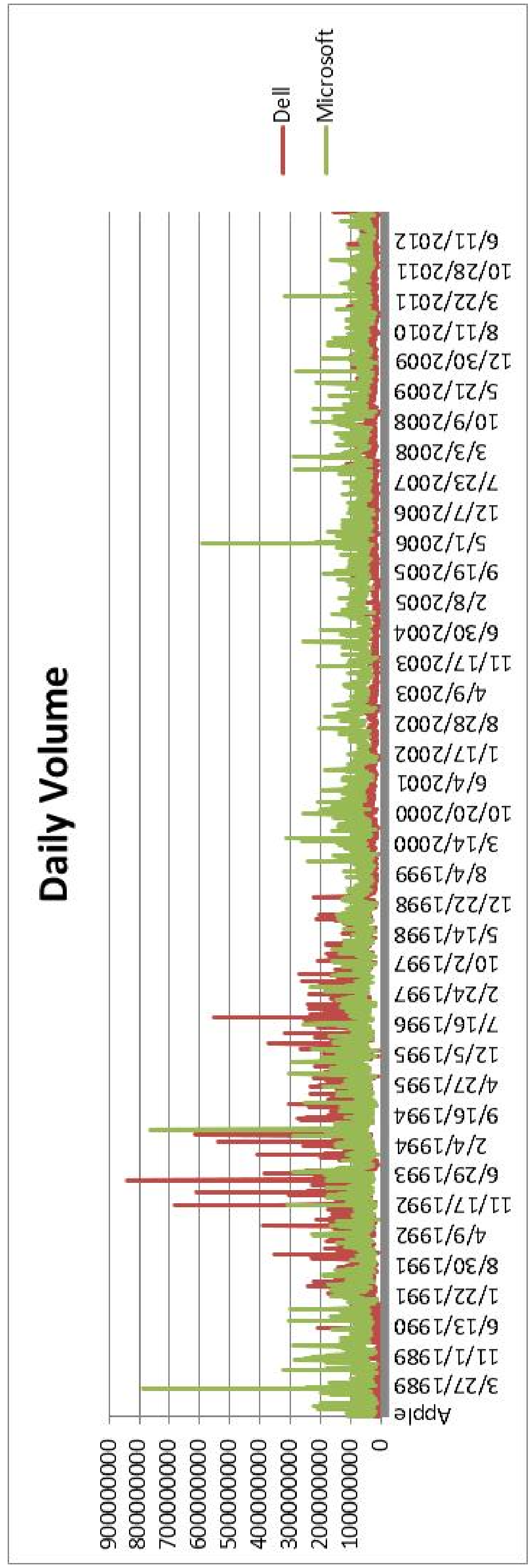
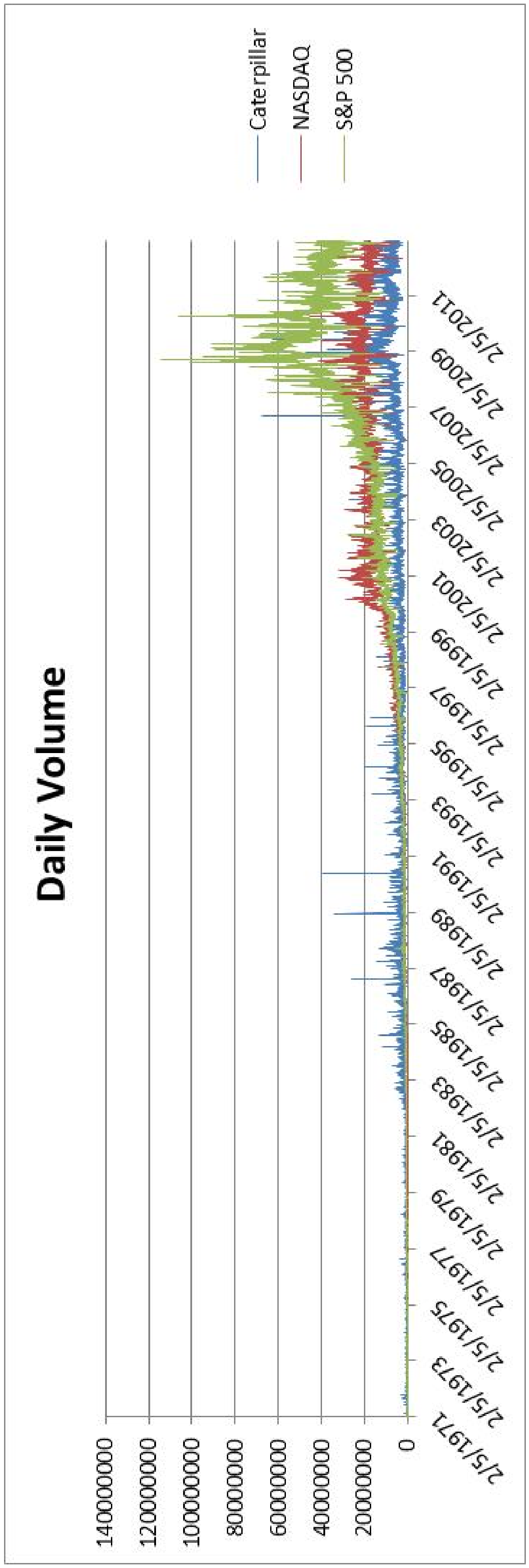
8/17/2010
8/17/2011
8/17/2012


Daily Volume
Apple
Microsoft
900000000
800000000
700000000
600000000
500000000
400000000
300000000
200000000
100000000
0
Apple
3/27/1989
11/1/1989
6/13/1990
1/22/1991
8/30/1991
4/9/1992
11/17/1992
6/29/1993
2/4/1994
9/16/1994
4/27/1995
12/5/1995
7/16/1996
2/24/1997
10/2/1997
5/14/1998
12/22/1998
8/4/1999
3/14/2000
10/20/2000
6/4/2001
1/17/2002
8/28/2002
4/9/2003
11/17/2003
6/30/2004
2/8/2005
9/19/2005
5/1/2006
12/7/2006
7/23/2007
3/3/2008
10/9/2008
5/21/2009
12/30/2009
8/11/2010
3/22/2011
10/28/2011
6/11/2012


Daily Open
Dell (DELL)
Microsoft (MSFT)


Daily Volume
Dell
Microsoft


Daily Volume
Caterpillar
NASDAQ
S&P 500
140000000
120000000
100000000
80000000
60000000
40000000
20000000
0
2/5/1971
2/5/1973
2/5/1975
2/5/1977
2/5/1979
2/5/1981
2/5/1983
2/5/1985
2/5/1987
2/5/1989
2/5/1991
2/5/1993
2/5/1995
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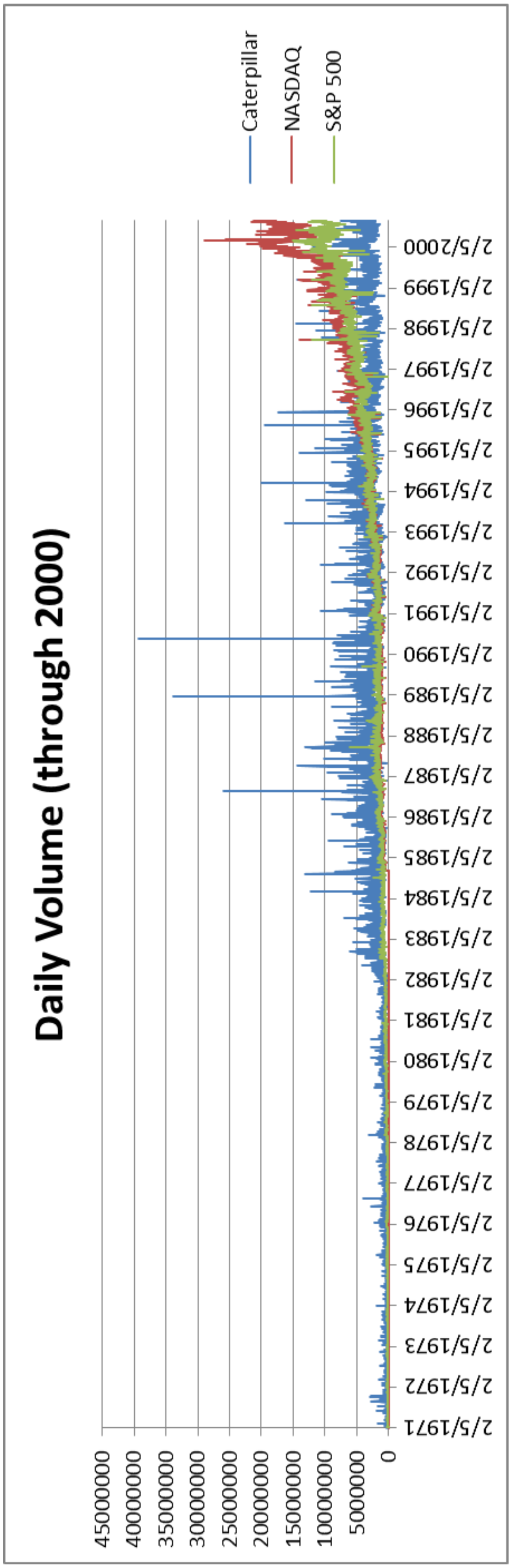
2/5/2007
2/5/2009
2/5/2011


Daily Volume (through 2000)
Caterpillar
NASDAQ
S&P 500
45000000
40000000
35000000
30000000
25000000
20000000
15000000
10000000
5000000
0
2/5/1971
2/5/1972
2/5/1973
2/5/1974
2/5/1975
2/5/1976
2/5/1977
2/5/1978
2/5/1979
2/5/1980
2/5/1981
2/5/1982
2/5/1983
2/5/1984
2/5/1985
2/5/1986
2/5/1987
2/5/1988
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2/5/1990
2/5/1991
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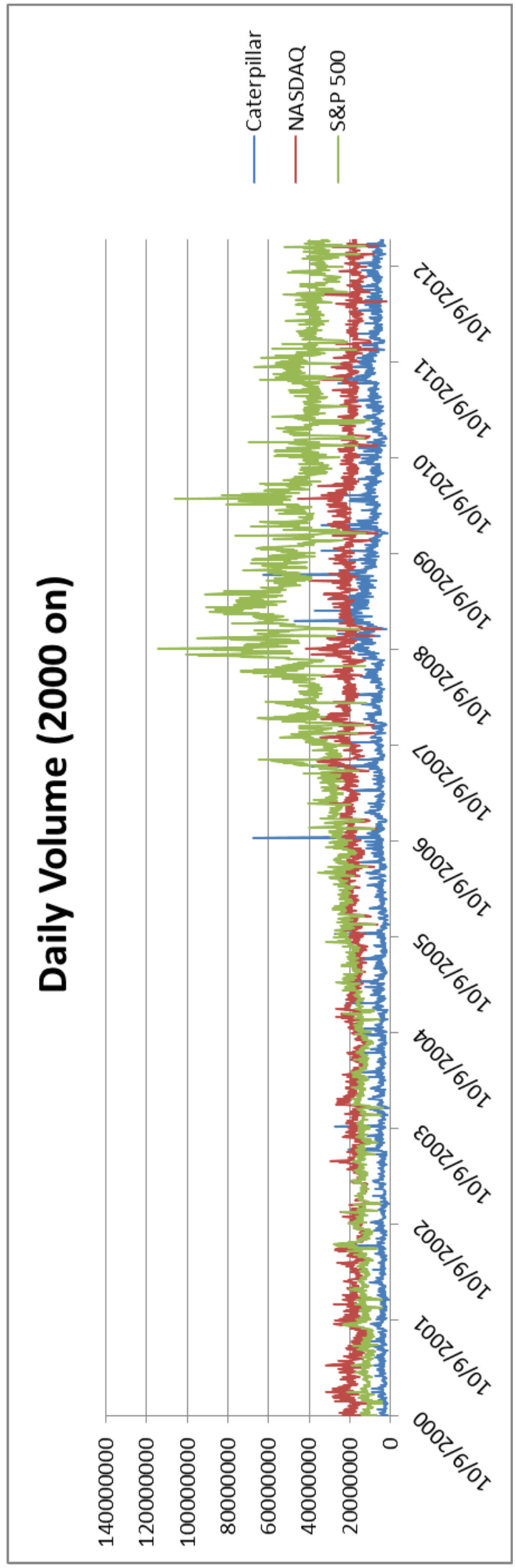
2/5/1999
2/5/2000


Daily Volume (2000 on)
Caterpillar
NASDAQ
S&P 500
140000000
120000000
100000000
80000000
60000000
40000000
20000000
0
10/9/2000
10/9/2001
10/9/2002
10/9/2003
10/9/2004
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10/9/2008
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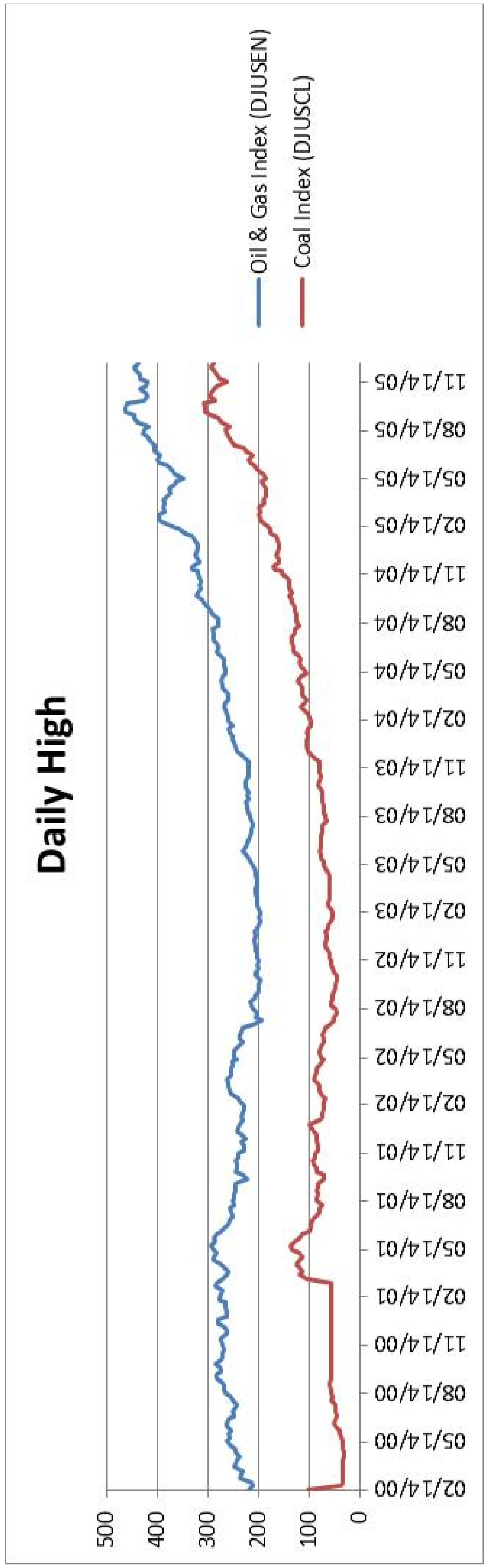
10/9/2010
10/9/2011
10/9/2012


Daily High
Oil & Gas Index (DJUSEN)
Coal Index (DJUSCL)
500
400
300
200
100
0
02/14/00
05/14/00
08/14/00
11/14/00
02/14/01
05/14/01
08/14/01
11/14/01
02/14/02
05/14/02
08/14/02
11/14/02
02/14/03
05/14/03
08/14/03
11/14/03
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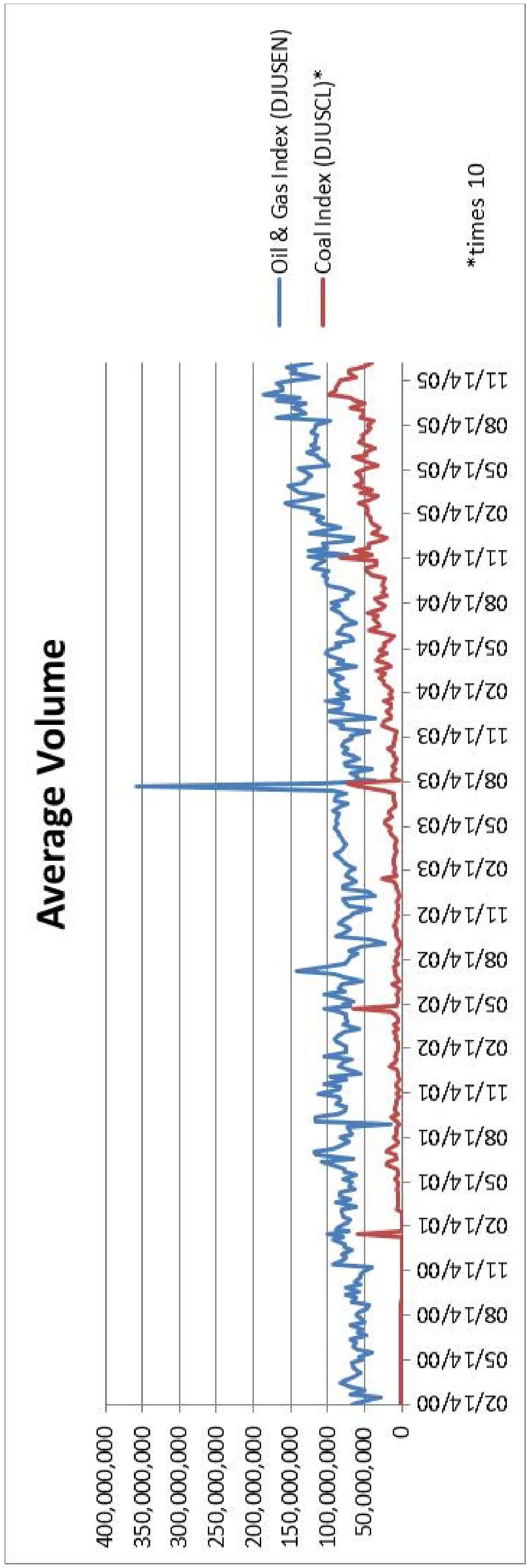
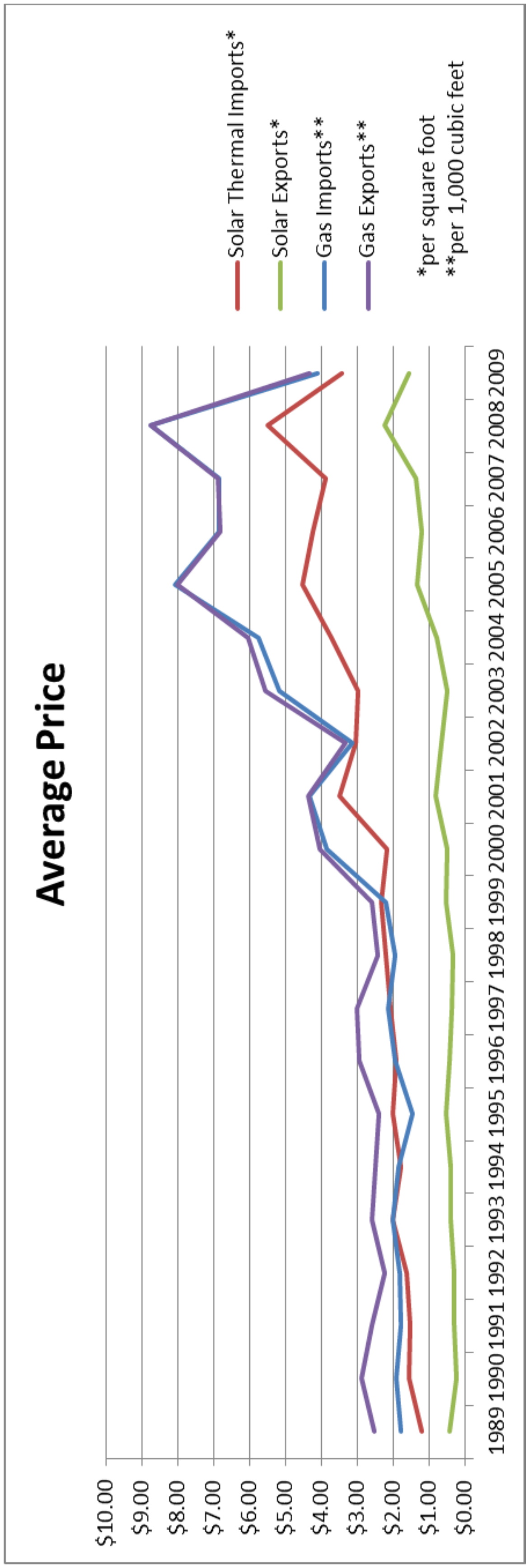
02/14/05
05/14/05
08/14/05
11/14/05


Average Volume
400,000,000
350,000,000
300,000,000
250,000,000
200,000,000
150,000,000
100,000,000
50,000,000
0
02/14/00
05/14/00
08/14/00
11/14/00
02/14/01
05/14/01
08/14/01
11/14/01
02/14/02
05/14/02
08/14/02
11/14/02
02/14/03
05/14/03
08/14/03
11/14/03
02/14/04
05/14/04
08/14/04
11/14/04
02/14/05
05/14/05
08/14/05
11/14/05
Oil & Gas Index (DJUSEN)
Coal Index (DJUSCL)*
*times 10


Average Price
Solar Thermal Imports*
Solar Exports*
Gas Imports**
Gas Exports**
*per square foot
**per 1,000 cubic feet
$10.00
$9.00
$8.00
$7.00
$6.00
$5.00
$4.00
$3.00
$2.00
$1.00
$0.00
1989 1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008 2009

APPENDIX B: REGRESSIONS

SUMMARY OUTPUT	Target (Prey)							
Regression Statistics			b =	0.03006				
Multiple R	0.0292		p =	0.00000				
R Square	0.0009							
Adjusted R Square	-0.0040							
Standard Error	0.2444							
Observations	206							
ANOVA								
	df	*SS*	*MS*	*F*	*Significance F*			
Regression	1	0.0104	0.0104	0.1740	0.6770			
Residual	204	12.1823	0.0597					
Total	205	12.1926						
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.0301	0.0633	0.4749	0.6354	-0.0948	0.1549	-0.0948	0.1549
X Variable 1	0.0000	0.0000	-0.4171	0.6770	0.0000	0.0000	0.0000	0.0000

SUMMARY OUTPUT	Walmart (Predator)							
Regression Statistics			d =	0.00000				
Multiple R	0.0429		r =	0.01255				
R Square	0.0018							
Adjusted R Square	-0.0031							
Standard Error	0.1618							
Observations	206							
ANOVA								
	df	*SS*	*MS*	*F*	*Significance F*			
Regression	1	0.0098	0.0098	0.3757	0.5406			
Residual	204	5.3375	0.0262					
Total	205	5.3473						
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-0.0125	0.0214	-0.5859	0.5586	-0.0548	0.0297	-0.0548	0.0297
X Variable 1	0.0000	0.0000	0.6130	0.5406	0.0000	0.0000	0.0000	0.0000

Regression for a one-predator, one-prey model for Target (Prey) and Walmart (Predator),

5/2/1983-4/2/2001, based on monthly stock volume

SUMMARY OUTPUT	S&P 500 (Prey)							
Regression Statistics			a =	0.0130				
Multiple R	0.1020		b =	0.0000				
R Square	0.0104		c =	0.0000				
Adjusted R Square	0.0006							
Standard Error	0.0691							
Observations	206							
ANOVA								
	df	*SS*	*MS*	*F*	*Significance F*			
Regression	2	0.0102	0.0051	1.0664	0.3462			
Residual	203	0.9691	0.0048					
Total	205	0.9793						
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.0130	0.0180	0.7213	0.4716	-0.0225	0.0484	-0.0225	0.0484
X Variable 1	0.0000	0.0000	-0.7258	0.4688	0.0000	0.0000	0.0000	0.0000
X Variable 2	0.0000	0.0000	1.4424	0.1507	0.0000	0.0000	0.0000	0.0000

Regression for a two-predator, one-prey model for S&P 500 (Prey), Walmart (Predator 1), and Target (Predator 2), 5/2/1983-4/2/2001, based on monthly stock volume (continued on next page)

SUMMARY OUTPUT	Walmart (Predator 1)							
Regression Statistics			e =	0.0013				
Multiple R	0.0002		d =	0.0000				
R Square	0.0000							
Adjusted R Square	-0.0049							
Standard Error	0.1619							
Observations	206							
ANOVA								
	df	*SS*	*MS*	*F*	*Significance F*			
Regression	1	0.0000	0.0000	0.0000	0.9973			
Residual	204	5.3473	0.0262					
Total	205	5.3473						
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-0.00134	0.0184	-0.0726	0.9422	-0.0376	0.0349	-0.0376	0.0349
X Variable 1	0.00000	0.0000	-0.0034	0.9973	0.0000	0.0000	0.0000	0.0000

SUMMARY OUTPUT	Target (Predator 2)							
Regression Statistics			g =	-0.0022				
Multiple R	0.0079		f =	0.0000				
R Square	0.0001							
Adjusted R Square	-0.0048							
Standard Error	0.2445							
Observations	206							
ANOVA								
	df	*SS*	*MS*	*F*	*Significance F*			
Regression	1	0.0008	0.0008	0.0127	0.9105			
Residual	204	12.1919	0.0598					
Total	205	12.1926						
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.00216	0.0278	0.0779	0.9380	-0.0526	0.0569	-0.0526	0.0569
X Variable 1	0.00000	0.0000	0.1126	0.9105	0.0000	0.0000	0.0000	0.0000

(continued from previous page) Regression for a two-predator, one-prey model for S&P 500 (Prey), Walmart (Predator 1), and Target (Predator 2), 5/2/1983-4/2/2001, based on monthly stock volume

SUMMARY OUTPUT	Target (Prey)							
Regression Statistics			b =	0.14289				
Multiple R	0.0489		p =	0.00000				
R Square	0.0024							
Adjusted R Square	-0.0269							
Standard Error	1.2289							
Observations	36							
ANOVA								
	df	*SS*	*MS*	*F*	*Significance F*			
Regression	1	0.1232	0.1232	0.0816	0.7769			
Residual	34	51.3435	1.5101					
Total	35	51.4667						
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.1429	0.2053	0.6959	0.4912	-0.2744	0.5602	-0.2744	0.5602
X Variable 1	0.0000	0.0000	0.2856	0.7769	0.0000	0.0000	0.0000	0.0000

SUMMARY OUTPUT	Walmart (Predator)							
Regression Statistics			d =	0.0000				
Multiple R	0.0085		r =	-0.1041				
R Square	0.0001							
Adjusted R Square	-0.0293							
Standard Error	4.2984							
Observations	36							
ANOVA								
	df	*SS*	*MS*	*F*	*Significance F*			
Regression	1	0.0455	0.0455	0.0025	0.9607			
Residual	34	628.1978	18.4764					
Total	35	628.2434						
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.1041	0.7178	0.1450	0.8856	-1.3547	1.5629	-1.3547	1.5629
X Variable 1	0.0000	0.0000	-0.0497	0.9607	0.0000	0.0000	0.0000	0.0000

Regression for a one-predator, one-prey model for Target (Prey) and Walmart (Predator), 5/1/2009-4/2/2012, based on detrended monthly stock volume

SUMMARY OUTPUT		S&P 500 (Prey)						
Regression Statistics			a =	0.2679				
Multiple R	0.2193		b =	0.0000				
R Square	0.0481		c =	0.0000				
Adjusted R Square	-0.0096							
Standard Error	3.4217							
Observations	36							
ANOVA								
	df	*SS*	*MS*	*F*	*Significance F*			
Regression	2	19.5176	9.7588	0.8335	0.4435			
Residual	33	386.3544	11.7077					
Total	35	405.8720						
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.2679	0.5720	0.4683	0.6426	-0.8958	1.4315	-0.8958	1.4315
X Variable 1	0.0000	0.0000	-1.0295	0.3107	0.0000	0.0000	0.0000	0.0000
X Variable 2	0.0000	0.0000	1.1973	0.2397	0.0000	0.0000	0.0000	0.0000

Regression for a two-predator, one-prey model for S&P 500 (Prey), Walmart (Predator 1), and Target (Predator 2), 5/1/2009-4/2/2012, based on detrended monthly stock volume (continued on next page)

SUMMARY OUTPUT	Walmart (Predator 1)							
Regression Statistics			e =	-0.0527				
Multiple R	0.1749		d =	0.0000				
R Square	0.0306							
Adjusted R Square	0.0021							
Standard Error	4.2323							
Observations	36							
ANOVA								
	df	*SS*	*MS*	*F*	*Significance F*			
Regression	1	19.2237	19.2237	1.0732	0.3075			
Residual	34	609.0196	17.9123					
Total	35	628.2434						
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.0527	0.7073	0.0744	0.9411	-1.3847	1.4900	-1.3847	1.4900
X Variable 1	0.0000	0.0000	-1.0360	0.3075	0.0000	0.0000	0.0000	0.0000

SUMMARY OUTPUT								
Regression Statistics			g =	-0.1469				
Multiple R	0.0926		f =	0.0000				
R Square	0.0086							
Adjusted R Square	-0.0206							
Standard Error	1.2250							
Observations	36							
ANOVA								
	df	*SS*	*MS*	*F*	*Significance F*			
Regression	1	0.4416	0.4416	0.2943	0.5910			
Residual	34	51.0250	1.5007					
Total	35	51.4667						
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.14689	0.2047	0.7175	0.4780	-0.2692	0.5629	-0.2692	0.5629
X Variable 1	0.00000	0.0000	0.5425	0.5910	0.0000	0.0000	0.0000	0.0000

(continued from previous page) Regression for a two-predator, one-prey model for S&P 500 (Prey), Walmart (Predator 1), and Target (Predator 2), 5/1/2009-4/2/2012, based on detrended monthly stock volume

SUMMARY OUTPUT	Dell (Prey)							
Regression Statistics			b =	0.0815				
Multiple R	0.0911		p =	0.0000				
R Square	0.0083							
Adjusted R Square	-0.0023							
Standard Error	0.2378							
Observations	96							
ANOVA								
	df	*SS*	*MS*	*F*	*Significance F*			
Regression	1	0.0445	0.0445	0.7863	0.3775			
Residual	94	5.3160	0.0566					
Total	95	5.3605						
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.0815	0.0977	0.8344	0.4062	-0.1124	0.2755	-0.1124	0.2755
X Variable 1	0.0000	0.0000	-0.8867	0.3775	0.0000	0.0000	0.0000	0.0000

SUMMARY OUTPUT	Microsoft (Predator)							
Regression Statistics			d =	0.0000				
Multiple R	0.0973		r =	0.0488				
R Square	0.0095							
Adjusted R Square	-0.0011							
Standard Error	0.1598							
Observations	96							
ANOVA								
	df	*SS*	*MS*	*F*	*Significance F*			
Regression	1	0.0230	0.0230	0.8991	0.3454			
Residual	94	2.4006	0.0255					
Total	95	2.4236						
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-0.0488	0.0530	-0.9212	0.3593	-0.1541	0.0564	-0.1541	0.0564
X Variable 1	0.0000	0.0000	0.9482	0.3454	0.0000	0.0000	0.0000	0.0000

Regression for a one-predator, one-prey model for Dell (Prey) and Microsoft (Predator),

1/2/1991-12/1/1998, based on monthly stock volume

SUMMARY OUTPUT	Dell (Prey)							
Regression Statistics			b =	-0.0224				
Multiple R	0.0778		p =	0.0000				
R Square	0.0060							
Adjusted R Square	-0.0033							
Standard Error	0.1450							
Observations	108							
ANOVA								
	df	*SS*	*MS*	*F*	*Significance F*			
Regression	1	0.0136	0.0136	0.6448	0.4238			
Residual	106	2.2286	0.0210					
Total	107	2.2422						
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-0.0224	0.0286	-0.7850	0.4342	-0.0791	0.0342	-0.0791	0.0342
X Variable 1	0.0000	0.0000	0.8030	0.4238	0.0000	0.0000	0.0000	0.0000

SUMMARY OUTPUT	Apple (Predator)							
Regression Statistics			d =	0.0000				
Multiple R	0.0401		r =	-0.0433				
R Square	0.0016							
Adjusted R Square	-0.0078							
Standard Error	0.2641							
Observations	108							
ANOVA								
	df	*SS*	*MS*	*F*	*Significance F*			
Regression	1	0.0119	0.0119	0.1706	0.6804			
Residual	106	7.3922	0.0697					
Total	107	7.4041						
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.0433	0.0839	0.5158	0.6071	-0.1230	0.2095	-0.1230	0.2095
X Variable 1	0.0000	0.0000	-0.4131	0.6804	0.0000	0.0000	0.0000	0.0000

Regression for a one-predator, one-prey model for Dell (Prey) and Apple (Predator), 1/2/1991-12/1/1998, based on monthly stock volume

SUMMARY OUTPUT	Microsoft (Prey)							
Regression Statistics			b =	-0.0273				
Multiple R	0.1059		p =	0.0000				
R Square	0.0112							
Adjusted R Square	0.0019							
Standard Error	0.1572							
Observations	108							
ANOVA								
	df	*SS*	*MS*	*F*	*Significance F*			
Regression	1	0.0297	0.0297	1.2017	0.2755			
Residual	106	2.6204	0.0247					
Total	107	2.6501						
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-0.0273	0.0310	-0.8821	0.3797	-0.0888	0.0341	-0.0888	0.0341
X Variable 1	0.0000	0.0000	1.0962	0.2755	0.0000	0.0000	0.0000	0.0000

SUMMARY OUTPUT	Apple (Predator)							
Regression Statistics			d =	0.0000				
Multiple R	0.2113		r =	-0.2420				
R Square	0.0447							
Adjusted R Square	0.0357							
Standard Error	0.2583							
Observations	108							
ANOVA								
	df	*SS*	*MS*	*F*	*Significance F*			
Regression	1	0.3307	0.3307	4.9556	0.0281			
Residual	106	7.0734	0.0667					
Total	107	7.4041						
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.2420	0.1070	2.2609	0.0258	0.0298	0.4542	0.0298	0.4542
X Variable 1	0.0000	0.0000	-2.2261	0.0281	0.0000	0.0000	0.0000	0.0000

Regression for a one-predator, one-prey model for Microsoft (Prey) and Apple (Predator),

1/3/2000-12/1/2008, based on monthly stock volume

SUMMARY OUTPUT	S&P 500 (Prey)							
Regression Statistics			b =	-0.0439				
Multiple R	0.1119		p =	0.0000				
R Square	0.0125							
Adjusted R Square	0.0085							
Standard Error	0.1406							
Observations	248							
ANOVA								
	df	*SS*	*MS*	*F*	*Significance F*			
Regression	1	0.0616	0.0616	3.1180	0.0787			
Residual	246	4.8598	0.0198					
Total	247	4.9214						
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-0.0439	0.0256	-1.7160	0.0874	-0.0942	0.0065	-0.0942	0.0065
X Variable 1	0.0000	0.0000	1.7658	0.0787	0.0000	0.0000	0.0000	0.0000

SUMMARY OUTPUT	Caterpillar (Predator)							
Regression Statistics			d =	0.0000				
Multiple R	0.0380		r =	-0.0441				
R Square	0.0014							
Adjusted R Square	-0.0026							
Standard Error	0.2460							
Observations	248							
ANOVA								
	df	*SS*	*MS*	*F*	*Significance F*			
Regression	1	0.0215	0.0215	0.3553	0.5517			
Residual	246	14.8880	0.0605					
Total	247	14.9095						
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.0441	0.0762	0.5782	0.5637	-0.1060	0.1941	-0.1060	0.1941
X Variable 1	0.0000	0.0000	-0.5961	0.5517	0.0000	0.0000	0.0000	0.0000

Regression for a one-predator, one-prey model for the S&P 500 (Prey) and Caterpillar (Predator), 1/1/2001-12/31/2001, based on daily stock volume

SUMMARY OUTPUT	NASDAQ (Prey)							
Regression Statistics			b =	-0.0404				
Multiple R	0.0953		p =	0.0000				
R Square	0.0091							
Adjusted R Square	0.0050							
Standard Error	0.1445							
Observations	248							
ANOVA								
	df	*SS*	*MS*	*F*	*Significance F*			
Regression	1	0.0470	0.0470	2.2530	0.1346			
Residual	246	5.1371	0.0209					
Total	247	5.1841						
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-0.0404	0.0263	-1.5359	0.1259	-0.0921	0.0114	-0.0921	0.0114
X Variable 1	0.0000	0.0000	1.5010	0.1346	0.0000	0.0000	0.0000	0.0000

SUMMARY OUTPUT	Caterpillar (Predator)							
Regression Statistics			d =	0.0000				
Multiple R	0.0261		r =	-0.0297				
R Square	0.0007							
Adjusted R Square	-0.0034							
Standard Error	0.2461							
Observations	248							
ANOVA								
	df	*SS*	*MS*	*F*	*Significance F*			
Regression	1	0.0102	0.0102	0.1678	0.6824			
Residual	246	14.8994	0.0606					
Total	247	14.9095						
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.0297	0.0750	0.3953	0.6929	-0.1181	0.1774	-0.1181	0.1774
X Variable 1	0.0000	0.0000	-0.4096	0.6824	0.0000	0.0000	0.0000	0.0000

Regression for a one-predator, one-prey model for the NASDAQ Composite(Prey) and Caterpillar (Predator), 1/1/2001-12/31/2001, based on daily stock volume

SUMMARY OUTPUT	Oil (Prey)							
Regression Statistics			b =	0.0198				
Multiple R	0.0695		p =	0.0000				
R Square	0.0048							
Adjusted R Square	0.0016							
Standard Error	0.2097							
Observations	308							
ANOVA								
	df	*SS*	*MS*	*F*	*Significance F*			
Regression	1	0.0653	0.0653	1.4841	0.2241			
Residual	306	13.4589	0.0440					
Total	307	13.5242						
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.0198	0.0161	1.2270	0.2208	-0.0119	0.0515	-0.0119	0.0515
X Variable 1	0.0000	0.0000	-1.2182	0.2241	0.0000	0.0000	0.0000	0.0000

SUMMARY OUTPUT	Coal (Predator)							
Regression Statistics			d =	0.0008				
Multiple R	0.0105		r =	61,188.92				
R Square	0.0001							
Adjusted R Square	-0.0032							
Standard Error	2390697.9261							
Observations	308							
ANOVA								
	df	*SS*	*MS*	*F*	*Significance F*			
Regression	1	192,926,297,952.25	192,926,297,952.25	0.0338	0.8543			
Residual	306	1,748,923,591,532,210.00	5,715,436,573,634.67					
Total	307	1,749,116,517,830,160.00						
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-61,188.9160	399,537.34	-0.1531	0.8784	-847,377.2058	724,999.37	-847,377.2058	724,999.37
X Variable 1	0.0008	0.0043	0.1837	0.8543	-0.0077	0.0093	-0.0077	0.0093

Regression for a one-predator, one-prey model for Dow Jones U.S. Oil and Gas Index(Prey) and ten times the Dow Jones U.S. Coal Index (Predator), 2/21/2000-12/12/05, based on monthly average stock volume

SUMMARY OUTPUT	Natural Gas Price (Prey)							
Regression Statistics			b =	0.0884				
Multiple R	0.0740		p =	0.0047				
R Square	0.0055							
Adjusted R Square	-0.0530							
Standard Error	0.1580							
Observations	19							
ANOVA								
	df	*SS*	*MS*	*F*	*Significance F*			
Regression	1	0.0023	0.0023	0.0936	0.7634			
Residual	17	0.4244	0.0250					
Total	18	0.4267						
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.0884	0.0692	1.2782	0.2184	-0.0575	0.2343	-0.0575	0.2343
X Variable 1	-0.0047	0.0155	-0.3059	0.7634	-0.0374	0.0280	-0.0374	0.0280

SUMMARY OUTPUT	Solar Thermal Price (Predator)							
Regression Statistics			d =	-0.0116				
Multiple R	0.1461		r =	-0.0959				
R Square	0.0213							
Adjusted R Square	-0.0362							
Standard Error	0.0931							
Observations	19							
ANOVA								
	df	*SS*	*MS*	*F*	*Significance F*			
Regression	1	0.0032	0.0032	0.3708	0.5506			
Residual	17	0.1475	0.0087					
Total	18	0.1507						
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.0959	0.0572	1.6765	0.1119	-0.0248	0.2167	-0.0248	0.2167
X Variable 1	-0.0116	0.0191	-0.6090	0.5506	-0.0518	0.0286	-0.0518	0.0286

Regression for a one-predator, one-prey model for natural gas import price per 1,000 cubic feet (Prey) and solar thermal import price per square foot (Predator), 1989-2009